 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **APRIL 2012**

# MT 2906 - REAL ANALYSIS AND LINEAR ALGEBRA

Date : 26-04-2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

# Answer ALL Questions

1. (a) For any , prove that .

**(OR)**

(b) If  is a convergent series, then prove that . (5 marks)

(c) (i) If  and , then prove that .

(ii) Find  such that  and find the limit of . (10+ 5 marks)

**(OR)**

(d) (i) State and prove Leibnitz rule.

(ii) Prove that the series  is convergent. (10 + 5 marks)

2. (a) If the limit of *f*(*x*) as *x**a* exists, then prove that it is unique.

**(OR)**

(b) State and prove the Binomial theorem for any positive integer m. (5 marks)

(c) (i) Prove that the limit of the sum of two functions is the sum of the limits of the two functions.

(ii) Prove that the limit of the product of two functions is the product of their limits. (6 + 9 marks)

**(OR)**

(d) (i) State and prove Chain rule for differentiation.

(ii) Use inverse function theorem to find the derivative of . (10 + 5 marks)

3. (a) Define the Riemann integral of a function.

**(OR)**

(b) If *f* *R*[*a*, *b*], then prove that | *f* |  *R*[*a*, *b*]. (5 marks)

(c) (i) Prove that every continuous function on [a, b] is Riemann integrable.

(ii) Let *f*(*x*) = *x*2. For each *n*  *N*, Let  be the partition  of [0, 1]. Compute  and . (9 + 6 marks)

**(OR)**

(d) State and prove the First Fundamental theorem of Calculus. (15 marks)

4. (a) Prove that a system of linear equations in *n* unknowns *fi* = 0, *i* = 1, 2, ..., *m* are linearly dependent if and only if the rank *r* of the augmented matrix is less than the number *m* of equations.

**(OR)**

(b) Prove that a square matrix is singular if and only if its columns are linearly dependent. (5 marks)

(c) (i) If p vectors from a set of k vectors A1, A2, …, Ak where p < k are linearly dependent, then prove that all the vectors are linearly dependent.

(ii) Find the complete solution of the system of equations: x1 – x2 + 2x3 = 1; 2x1 + x2 – x3 = 2. (8 + 7 marks)

**(OR)**

(d) If a homogeneous linear system of m equations in n unknowns AX = 0 has rank r < n, then prove that all solutions may be written as linear combinations of n – r linearly independent solutions. Also prove that when r = n, the only solution is the dependent vector 0. (15 marks)

5. (a) Explain the process of reduction to diagonal form.

**(OR)**

(b) Write short notes on quadratic forms. Write down the matrix of the quadratic form *x*12 + *x*22 – 3*x*32 + 2*x*1*x*2 – 6*x*1*x*3. (5 marks)

(c) Find the characteristic roots and the associated space of characteristic vectors for the matrix A = .

**(OR)**

(d) Apply Gram-Schmidt process to the vectors {1, 2, 1, 1}, {1, -1, 0,2}, {2, 0, 1, 1) to find a set of orthonormal vectors. (15 marks)

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